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MOORING SYSTEMS FOR VERY LARGE FLOATING STRUCTURES

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ABSTRACT – Owing to scarcity of land, very large floating structures (VLFS) are now being designed to cater for the increase in population and growth of coastal areas. The applications of VLFS include floating piers, floating airports, floating bridges, floating fuel storage facilities and even floating cities. One of the key design aspects of VLFS is the mooring design. Mooring design of VLFS is a challenge due to huge size of the structures, environmental loads, shallow water depths, space constraint for mooring lines and anchor installation. There are additional challenges pertaining to transportation of blocks, integration onsite and design allowance for possible future expansion of the VLFS. This paper examines the hydrodynamic and mooring design of a typical VLFS. The relevant concepts, motion response, mooring design and design criteria will be presented. The mooring design will incorporate sensitivity studies on different material choices for mooring lines. Chains, wire ropes and polyester (Dyneema) will be considered for the mooring design. The chain mooring system is compared with piles mooring system. Additional issues pertaining to installation and future expansion of VLFS will be discussed.

KEYWORDS – *Very Large Floating Structures, VLFS, Mooring, Hydrodynamics, Chains, Piles*

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1. INTRODUCTION

There are multiple ways for station keeping of VLFS. Mooring chains, tension legs, caisson dolphins and piles are some of the options for mooring system design of VLFS. The mooring options may be integrated with wave energy dissipation devices to minimize the complexity of the mooring system design. This paper examines two designs, namely, mooring chains and piles for station keeping of VLFS. For mooring chains, a time domain mooring analysis was carried out. The calculations are based on software calculations and can be computed in normal computers. Linear diffraction method was used to calculate the response amplitude operator's (RAO) and quadratic transfer function (QTF) for the VLFS. The hydrodynamic analysis was carried out using Hydrostar software. The mooring analysis was carried out by using Ariane and Orcaflex software. Pile sizing was carried out based on total environment loads (from mooring analysis). The two mooring options were then compared based on weight, location, seabed and water depth.

2. MOORING SYSTEM WITH CHAINS

2.1 VLFS Hydrodynamics

The sections below provide the hydrodynamic analysis methodology, input data used for analysis and the results.

2.1.1 Analysis Methodology

The purpose of the hydrodynamic analysis is to evaluate motion and QTF RAOs for mooring analysis. The calculations are carried out by means of the Bureau Veritas computer program HYDROSTAR FOR EXPERT [4]. Based on the potential theory, this software predicts the three-dimensional flow of wave diffraction and radiation around floating or fixed bodies in deep water and water of finite depth. The singularity method of Kelvin's sources is used to solve the first order problems while Molin's method [4] is applied to evaluate the complete second order low-frequency and high-frequency wave loads.

Numerical results have shown that the Newman approximation [4] overestimates the drift loads for the extreme values. These values of drift loads are very important for the shallow water response. The Newman approximation is not efficient for this kind of study. Thus, full QTFs using near field formulation are calculated [4]. The hydrodynamic analysis follows the process represented by the scheme on Fig. 1. The hydrodynamic model is composed of flat panels representing the geometry of the submerged part of the hull.

2.1.2 MODEL Characteristics

The main characteristics of the VLFS used for analysis are given in Table 1. The water depth at the VLFS location is 20m [4].

Table 1. VLFS Particulars

Vessel Type	VLFS
Length, L	500.0 m
Breadth, B	100.0 m
Depth, D	5.0 m
Mean Draft, T	2 m
Displacement, Δ	102,500 MT
Buoyant volume	100,000 m^3
LCG from aft perpendicular	250 m
VCG from baseline	1.32 m
Yaw radius of gyration	125.0 m
Pitch radius of gyration	125.0 m

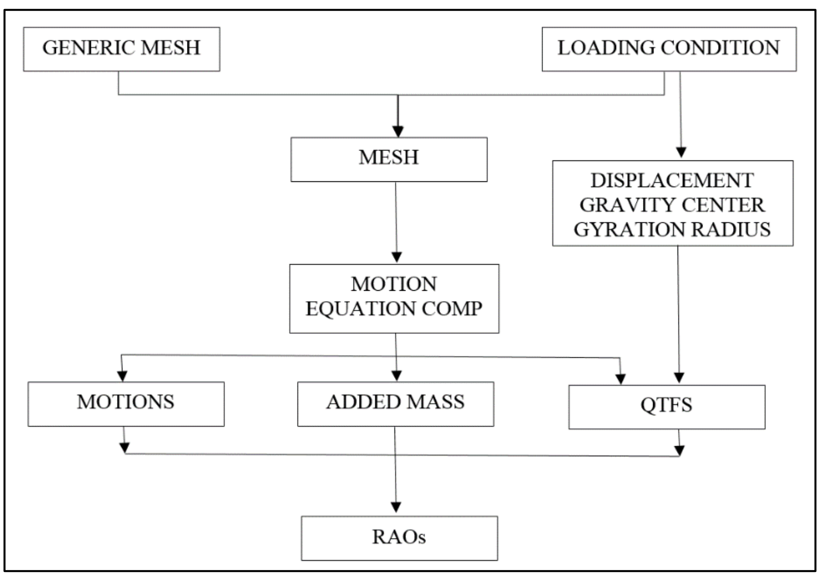


Fig. 1. Approach for Hydrodynamic Analysis

2.1.3 Hydrodynamic Mesh

The mesh model is shown in Fig. 2.

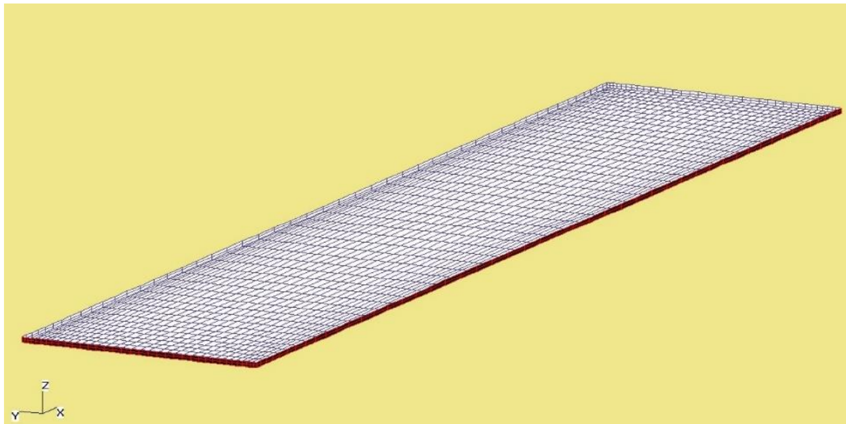


Fig. 2. VLFS Mesh

2.1.4 Roll Damping

The damping due to radiation is computed by HydroStar. However, in addition to the radiation damping, there are other sources of damping acting on the floating bodies such as the fluid viscosity and the mooring systems damping. The effects of viscosity on the hull and on the appendages on roll damping are generally higher than the radiation damping.

Hydrostar recommends Quadratic damping for barge shaped vessels [4]. In absence of model test results of VLFS, Quadratic damping [4] was considered for analysis. The equation for quadratic damping is given by

$$B_Q = \frac{1}{2}\rho C_D B^4 L \tag{1}$$

where B_Q is the quadratic damping, ρ the fluid density, B the breadth of VLFS, L the VLFS length and C_D = 0.1is coefficient [4].

Sensitivity analysis was carried out for different types of damping (linear & quadratic) inorder to assess the effect of damping on ROLL RAOs. The effect of different damping values (no damping, 5 – 10% linear damping, quadratic roll damping) on Roll RAO amplitude is shown in Fig. 3. The damping has a minor effect on the Roll RAO Amplitude.

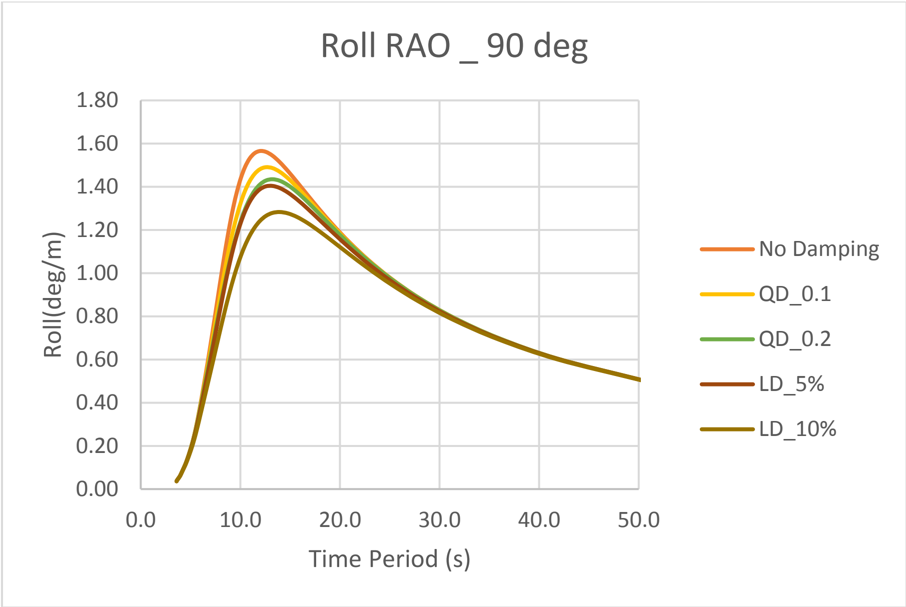


Fig. 3. Comparison between different roll damping values

2.1.5 Hydrodynamic Analysis Results

The plots of Heave, Roll & Pitch RAOs are shown in Figs. 4, 5 and 6, respectively.

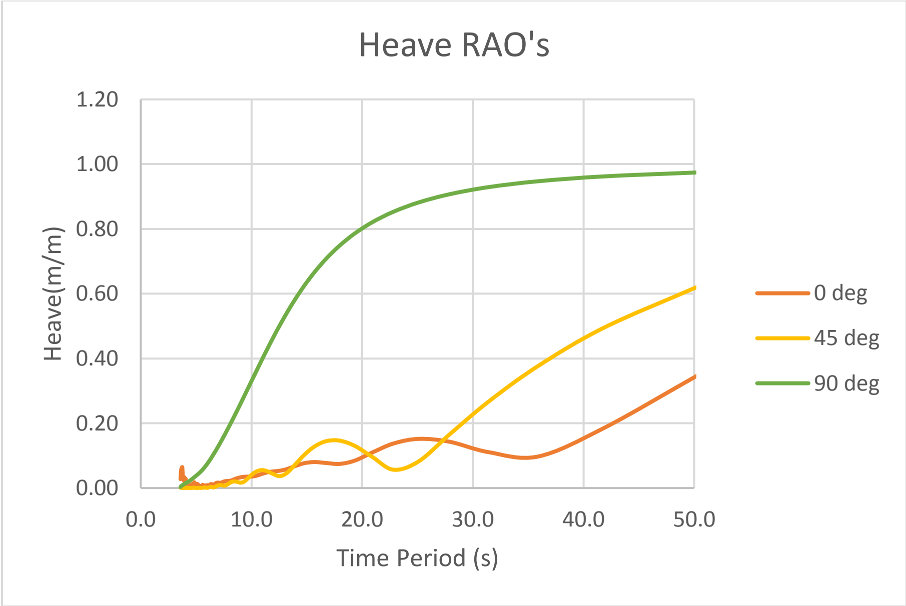


Fig. 4. Heave RAO's for head, quarter & beam seas

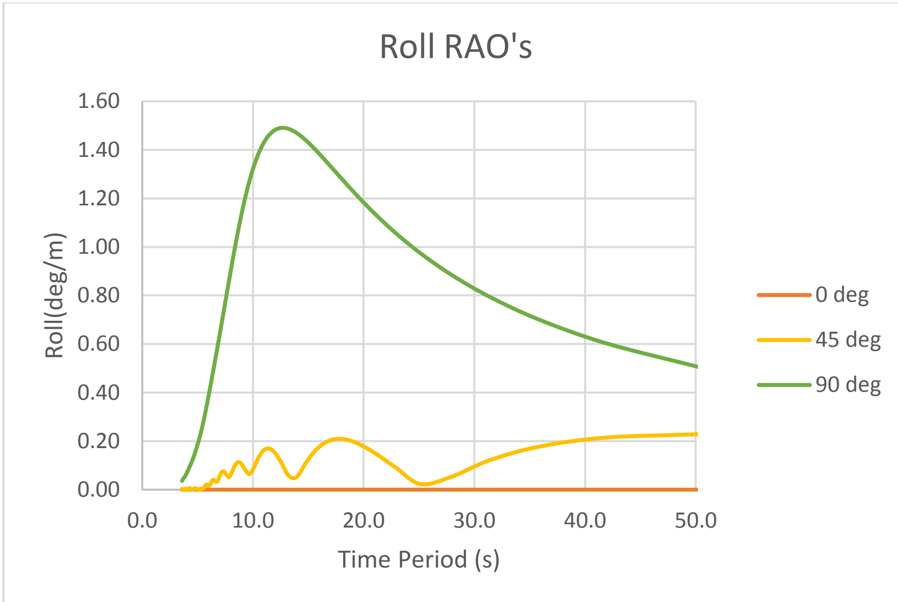


Fig. 5. Roll RAO's for head, quarter & beam seas

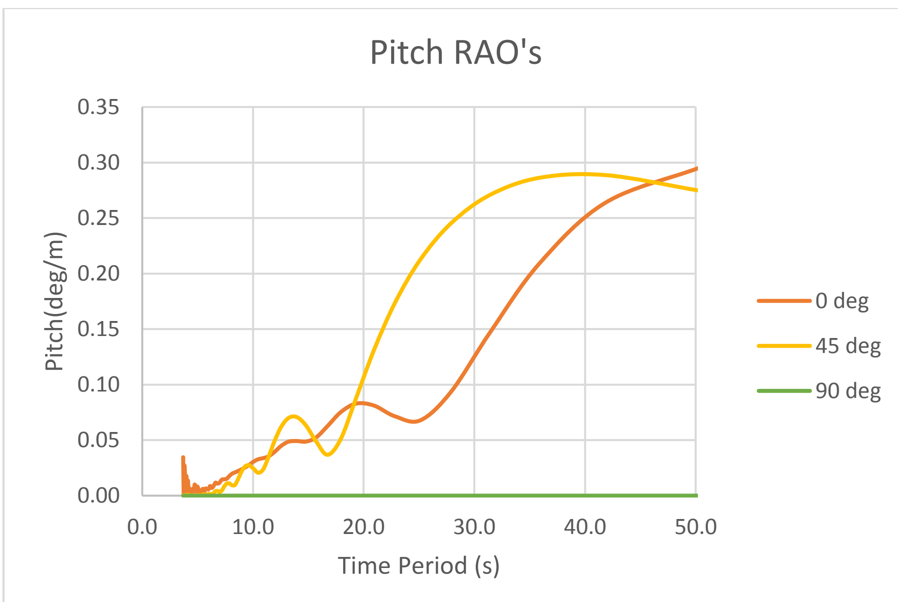


Fig. 6. Pitch RAO's for head, quarter & beam seas

Added Mass

Added mass under the specified conditions used for mooring analysis (horizontal components) are summarized in Table 2. They are evaluated at the vessel centre of gravity.

Table 2. Added Masses of VLFS

Component	Quantity
Surge	3.76E+06 kg
Sway	1.43E+07 kg
Sway/yaw	3.08E+03 kg.m
Yaw	2.32E+11 kg.m^2

2.2 MOORING ANALYSIS

The sections below provide the mooring analysis methodology, input data used for analysis and the results.

2.2.1 Mooring Analysis Methodology

For Mooring Analysis VLFS is considered as rigid body. Hydroelastic motion for VLFS was not considered. There are multiple ways to minimize the hydroelastic effect [5] and thus reduce the impact of hydroelastic motion on mooring loads. The methodology for the extreme strength analysis of the spread moored system is described below.

Quasi Dynamic Analysis

Quasi dynamic analysis is performed using the Ariane software. This is to evaluate the low frequency and wave frequency loads on the mooring system. The low frequency response of the vessel is evaluated by numerical resolution in time domain and at the end of each time step of this numerical integration, the wave frequency motions are added. Then the instantaneous tensions are derived from the tension-offset curves (characteristics). In order to achieve statistical significance, for each weather combination, 10800 seconds (3 hours) simulations were ran and the attached maximum tensions in the mooring system were determined. A ramp up period of 2000 seconds was used. For a given metocean state, 20 simulations were performed for randomly chosen seeds by software. These seeds correspond to sets of elementary wave components. The value of the design tension for the considered metocean state takes into account the dispersion of the maximum tensions in the lines due to the seed. It is based on the formula given by

$$T_D = T_{mean} + aT_\sigma \tag{2}$$

where T_{mean} is mean of the maximum tensions for the 20 simulations, T_σ is the standard deviation of the maximum tensions for the 20 simulations, a is a factor based on type of analysis and number of simulations (i.e. a = 0.5 for 20 simulations for quasi dynamic analysis [2] and a = 0.6 for 5 simulations for dynamic analysis [2])

Dynamic Analysis

Dynamic Analysis is performed for the worst load cases to evaluate the maximum tension in the most loaded Line (Quasi Dynamic Analysis). The Fairlead motion of the most loaded line is used as input in Orcaflex which then uses the time series fairlead motion for the analysis. Five simulations (for 5 random seed numbers) were ran for dynamic analysis. The value of the design tension for the considered metocean state takes into account the dispersion of the maximum tensions in the lines due to the seed. It is based on the formula given by Eq. (2).

The following steps are done to post process the results from the various runs for a particular load case.

- From all the maximum line tensions standard deviation is calculated →T_σ
- The mean of all the maximum mooring line tensions are calculated → T_{mean}.
- Design tension is computed from $T_D = T_{mean} + a^* T_\sigma$.
- T_D is used for Factors of Safety (FOS) calculation.
- Offset is obtained in the similar way as Tension.

SLF (Single Line Failure) Case

For SLF (Single Line Failure) the second most loaded line in the Intact Case will be broken to find the maximum tension. The most loaded line in the Intact Case will be broken to find the maximum offset. The environmental cases to be used in analysis will be the worst loading cases from Intact Analysis. The tension and offset will be calculated the same way as for the intact case.

2.2.2 Mooring Layout

The system is composed of VLFS and 8 mooring lines. The mooring layout is as shown in Fig. 7.

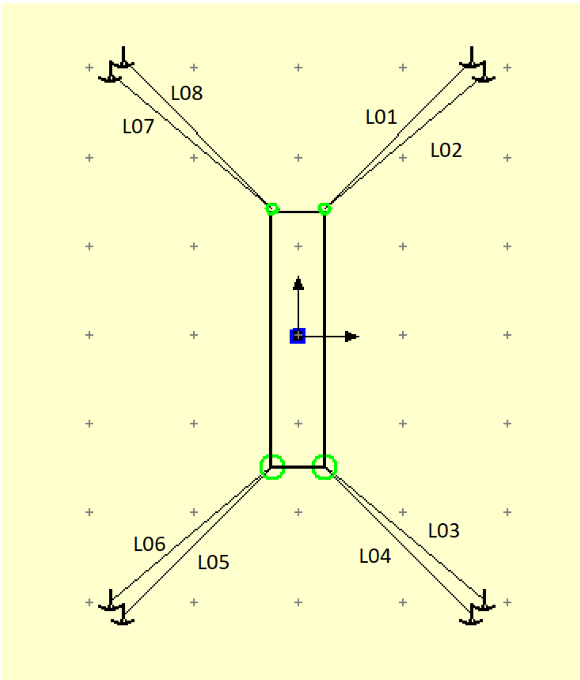


Fig. 7. Mooring layout

2.2.3 Vessel Damping

Additional linear damping was considered for the mooring analysis. The equations below provide the formula for the calculation of low frequency damping [2].

$$B_{xx} = 0.06\sqrt{K_{Oxx}(m + Ma_{xx})} \quad (3)$$

$$B_{yy} = 0.06\sqrt{K_{Oyy}(m + Ma_{yy})} \quad (4)$$

$$B\psi\psi = 0.10\sqrt{K_{O\varphi\varphi}\left[I_{\varphi\varphi} + Ma_{\varphi\varphi} + (m + Ma_{yy})x_G{}^2\right]} \quad (5)$$

where B_{xx} is the linear damping coefficient in surge, B_{yy} is the linear damping coefficient in sway, Bψψ is the linear damping coefficient in yaw, m is the mass, L is the length, B is the breadth, Ma_{xx}, Ma_{yy}, $Ma_{\varphi\varphi}$ refers to added mass of the vessel. *Koxx*, *Koyy* and *Koφφ* are the mooring system stiffness as calculated for average position of vessel for the most probable environment, x_G refers to vessel Longitudinal centre of gravity measured from centre of vessel and $I_{\varphi\varphi}$ refers to moment of inertia in yaw calculated at centre of gravity of the vessel.

The mooring system stiffness (*Koxx*, *Koyy* and *Koφφ*) have been calculated for equilibrium position of vessel and provided in Table 3. Table 4 indicates the damping coefficients calculated based on Eqs. (3), (4) and (5).

Table 3. Mooring System Stiffness

	K_{oxx} (kg/s^2)	K_{oyy} (kg/s^2)	$K_{o\varphi\varphi}$ (kg/s^2)
Mooring System Stiffness	2.16E+04	1.82E+04	9.43E+08

Table 4. Linear Damping coefficients

	B_{xx} (kg/s)	B_{yy} (kg/s)	$B_{\psi\psi}$ (kg.m^2/s)
Linear Damping Coefficients	9.08E+04	8.75E+04	4.16E+09

2.2.4 Mooring Lines Characteristics

Anchor radius is 350m and the total paid out length is 355m. Table 5 shows the line segments used for mooring analysis and sensitivity studies. The chain properties are shown in Tables 6. Table 7 provides the line segments minimum breaking loads (MBL).

Table 5. Line Segments

Line Type	Top Segment	Bottom Segment
Chain Only	Chain – 355m	
Wire + Chain	6-strand Wire – 50m	Chain – 305m
Dyneema Rope +Chain	Superline Dyneema – 50m	Chain – 305m

Table 6. Line Properties

Line Type	Value
Chain	76mm R3 studlink
Wire	76 mm 6-strand
Dyneema	76mm Superline

Table 7. Line MBL

Line Type	Value
Chain MBL (MT)	405.0
Wire MBL (MT)	378.0
Dyneema MBL (MT)	340.0

Note: Corrosion Allowance is 0.4mm / year [1]. Based on field life of 20 years Chain MBL is calculated for 76mm chain.

2.2.5 Design Criteria

Safety factors as per API guidelines [1] is shown in Table 8. The minimum ground length is positive to prevent anchor uplift.

Table 8. Mooring Line Safety Factor

Type of analysis	INTACT	SLF
Quasi Dynamic	2.00	1.43
Dynamic	1.67	1.25

2.2.6 Environmental Conditions

Table 9 shows the environmental data that is used for the mooring strength checks [3].

Table 9. Environment Data

Direction (from)	Wave			Wind	Current
	γ	Hs (m)	Tp (s)	(m/s)	(m/s)
Head	5	1	3.6	25	2.2
Quarter	5	1	3.6	25	2.2
Beam	3.3	0.6	3.1	25	1

The waves are modeled using JONSWAP spectrum. Note: (1) directional variation of wind was done for ± 45 degrees and (2) current was considered one by one for all 3 directions.

2.2.7 Load Cases

Intact load Cases

In order to find out the maximum loads and offsets, different environmental conditions were used in the analysis as shown in Table 13.

Table 10. Load Cases for mooring analysis

Wave	Wind	Current	Total Loadcases
0	0,-45,45	0,45,90	9
45	45,0,90	0,45,90	9
90	90,45,135	0,45,90	9
Total Load Cases			27

SLF Load Cases

For the worst Intact case SLF was run to find the maximum tension by breaking the second most loaded line and maximum offset by breaking the most loaded line.

2.2.8 Results

The results for the mooring analysis are presented in the sections below. Detailed results are presented in Appendix A. The results are presented in Tables 11 and 12.

Table 11. Intact Case Results

Env	Max Line Tension (MT)	Offset (m)
0	110.6	5.9
45	177.0	7.3
90	180.0	5.9

Table 12. SLF Case Results

Max Line Tension (MT)	Offset (m)
285.0	8.7

The line safety factors as shown in Table 13 are within the limits. The minimum ground length of chain for Intact case is 85m. The minimum ground length of chain for SLF case is 20m.

Table 13. Safety Factors

Analysis	Max Line Tension (MT)	Line MBL (MT)	Safety Factor
Intact	180.0	405.0	2.26 (>1.67)
SLF	285.0	405.0	1.42 (>1.25)

2.2.9 Sensitivity Analysis

Sensitivity Analysis was carried out with top section of line replaced with wire and then with dyneema rope. The analysis was carried out for the beam direction for which maximum line tension occurred. The results are shown in Tables 14 and 15. The mooring loads are similar to loads obtained in Table 13 for chain segments.

Table 14. Intact Case Results

Line Type	Intact Line Tension (MT)	VLFS Offset (m)
Wire Rope	169.7	5.2
Dyneema	161.2	5.0

Table 15. SLF Case Results

Line Type	SLF Line Tension (MT)	VLFS Offset (m)
Wire Rope	280.3	6.4
Dyneema	274.5	6.4

3. PILE MOORING SYSTEM

An alternative solution as compared to mooring lines would be to use steel piles to support the VLFS.

3.1 Methodology

The environmental loads (wind, wave and current) acting on the VLFS were calculated in Section 3. The piles were designed to withstand the environmental forces acting on VLFS and the piles structure. The piles are fixed on the seabed. Further analysis will be required based on detailed seabed profile for seabed reaction forces on piles. The VLFS will slide up and down the piles. There will be no axial compression force on Piles due to VLFS weight. Thus, bending stress will be the design criteria for sizing of Piles. The maximum allowable bending stress [6] is given in (4).

$$F_b = 0.75F_y \ \ for \ \frac{D}{t} \leq \frac{10{,}340}{F_y} = 47 \quad (4)$$

where the yield stress F_y = 220 MPA (mild steel) and F_b is the allowable bending stress on the pile.

The total bending moment acting on the piles was calculated and used to determine the diameter, thickness and number of piles required for station keeping of VLFS.

3.2 Pile Parameters

The dimensions of the pile are provided in Table 16.

Table 16. Pile Particulars

Parameters	Value
Length	20 m
Diameter, D1	1.25 m
Thickness, t	0.03 m
Section Modulus , S	0.034 m^3
L/r	32
D/t	41.7
Yield Stress (Fy)	220 MPa

3.3 Environmental Forces

The environmental forces acting on VLFS were calculated in Section 2 (mooring analysis) and the maximum wind, wave and current loads for head and beam seas are presented in Table17. The wave and current forces acting on pile have been calculated in Appendix B.

The environmental forces acting on VLFS and piles are provided in Table 17.

Table 17. Environmental Forces on VLFS and Single Pile

Environmental Forces	Head	Beam
VLFS (MT)	78.9	288.0
Single Pile (MT)	6.0	1.7

3.4 Calculation & Results

The bending Stress [6] is given by

$$F_b = \frac{BM}{S} \tag{5}$$

where *BM* is the bending moment (Force x Force arm) and *S* is the section modulus. The bending stress calculations for the piles are provided in Table 18.

Table 18. Bending Stress on Piles

Direction	Number of Piles	Total Force (MT)	Force Arm (m)	Bending Moment (kNm)	Bending Stress per Pile (MPa)	Allowable Stress (MPa)
Head	4	102.5	20	20147	147.1	165
Beam	12	304.4	20	60395	147.0	165

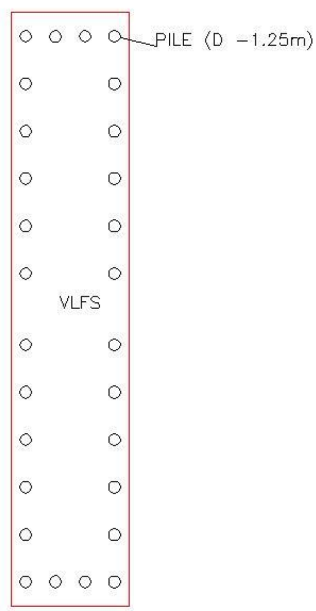


Fig. 8. VLFS station keeping using PILES

3.5 Comparison with Mooring Chains

The total number of piles required for station keeping of VLFS is 28. Table 19 provides a comparison of total weight of piles versus chains. The chains weight is much less than Piles weight.

Table 19. Piles versus Mooring Chain Weight

Mooring Component	Length	Number	Unit Weight (MT)	Total Weight (MT)
Piles	20	28	18.1	505.5
Chain System	355	8	56.9	455.3

Note that the length of piles excludes the length of pile below seabed. The additional length will lead to further increase in pile weight. Also, the chain system includes weight of chain and anchors.

4. INSTALLATION & FUTURE EXPANSION

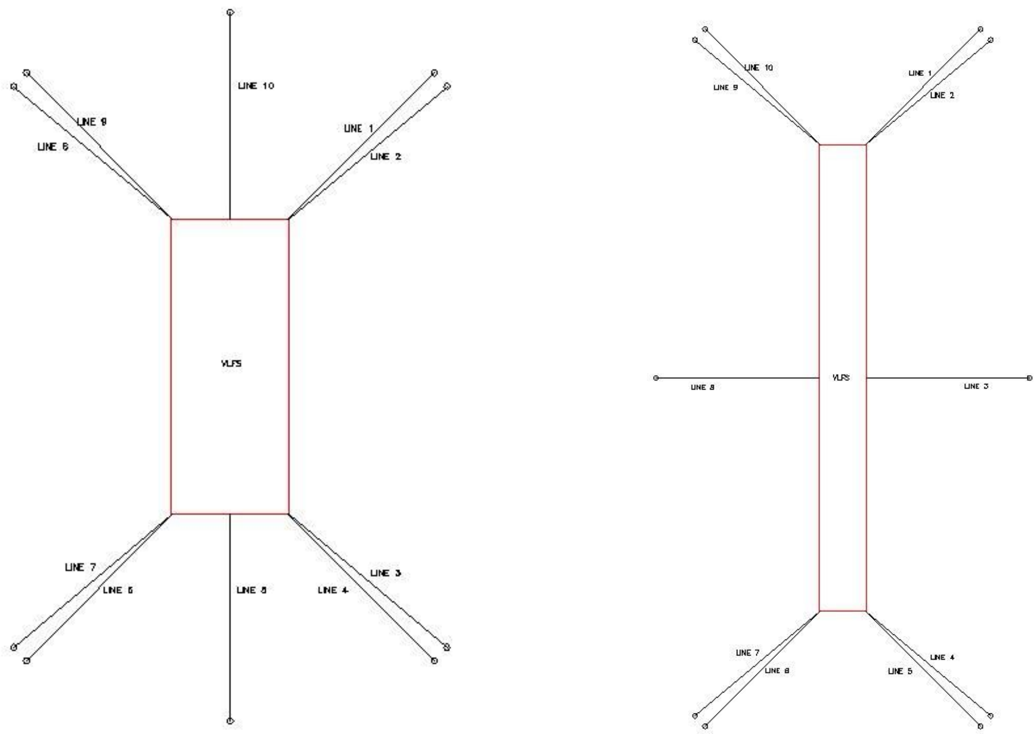
4.1 Installation

The VLFS can be towed in multiple blocks to the field and then assembled together. The mooring anchors and chain can be installed using Anchor Handling Tug (AHT). The lines pretension can be carried out using portable winches or Harbor tugs.

4.2 Future Expansion

The mooring lines are connected in bow and stern of VLFS. Future expansion can be carried out by adding additional blocks in front or side (Fig. 9).

Additional mooring lines will be required to handle environment due to increase in size. The existing mooring lines may be required to be relaid and re-connected depending on if additional blocks are added from side or from front.



a) 500m x 200m VLFS b) 1000m x 100m VLFS

Fig. 9. Mooring Layout for VLFS (Future Expansion)

5. CONCLUSION

The paper provides two mooring options for VLFS station keeping. The first option is to use mooring chains and anchors. The calculations are based on software calculations and can be computed in normal computers. This method allows a quick estimate of the mooring loads on the VLFS. Hydrodynamic Analysis was carried out for VLFS. Hydroelastic motion for VLFS was not considered. There are multiple ways to minimize the hydroelastic effect [5]. This allows diffraction software programs to analysze the hydrodynamic characteristics of VLFS. The VLFS mooring analysis was carried out for 8 point spread moored system. The mooring loads were within safety limits as shown in Table 13. Additional sensitivity analysis was carried out for different top sections of mooring lines (Wire & Dyneema). The mooring loads were close to chain only lines loads.

The second option was use of piles for stationkeeping of VLFS. A total of 28 piles were required for mooring system design of VLFS. The weight comparison of mooring chains and piles showed that chains are more effective for mooring design. Piles are quite sensitive to the soil characteristics and seabed profile. Drag Anchors are less sensitive and by adjusting the fluke angles they can be deployed in different types of soil (clay, sand). However, while chains require a mooring zone of 350m from VLFS edges, the piles can be placed within the VLFS effectively limiting the mooring zone to dimensions of VLFS.

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APPENDIX A – DETAILED RESULTS FOR MOORING ANALYSIS

Case	Wave	Wind	Current	L01	L02	L03	L04	L05	L06	L07	L08	offset
	(deg)	(deg)	(deg)	(MT)	(MT)	(MT)	(MT)	(MT)	(MT)	(MT)	(MT)	(m)
1	0	0	0	51.7	43.8	17.5	18.1	18.1	17.5	43.8	51.7	3.9
2	0	45	0	46.9	40.0	42.0	31.5	8.1	7.9	32.1	35.4	4.3
3	0	-45	0	38.5	33.9	23.5	25.3	25.3	23.5	33.9	38.5	3.2
4	0	0	45	67.3	96.4	58.8	69.4	25.6	23.1	6.1	6.4	5.4
5	0	45	45	78.3	110.6	77.4	89.2	22.2	20.0	5.9	6.3	5.7
6	0	-45	45	61.7	102.1	68.3	85.4	38.1	33.6	4.8	4.9	5.9
7	0	0	90	47.7	42.0	37.2	31.1	13.6	13.1	31.0	34.0	4.1
8	0	45	90	63.2	54.0	66.9	48.7	9.1	8.6	28.5	31.6	5.0
9	0	-45	90	33.7	33.8	37.7	40.0	23.3	21.8	21.0	22.2	3.4
10	45	45	0	117.1	91.3	136.8	73.3	7.8	7.5	74.6	84.6	6.5
11	45	90	0	125.1	94.4	144.3	77.6	7.5	7.1	78.3	90.7	6.7
12	45	0	0	86.0	70.7	85.9	56.7	14.3	15.0	74.9	81.0	5.4
13	45	45	45	134.5	171.0	137.6	132.9	35.0	30.4	36.4	42.8	6.7
14	45	90	45	142.1	177.4	158.9	154.0	39.1	33.4	34.3	40.0	6.8
15	45	0	45	111.0	150.4	110.9	118.0	42.0	36.5	28.6	32.9	6.4
16	45	45	90	166.3	124.9	164.4	100.1	16.5	14.9	101.3	121.1	7.3
17	45	90	90	158.0	117.7	177.2	107.7	15.1	13.8	102.6	121.9	7.3
18	45	0	90	150.3	113.6	140.3	82.2	20.1	18.9	104.0	121.5	6.9
19	90	90	0	94.0	93.7	115.2	75.0	6.6	6.6	35.9	37.1	5.3
20	90	135	0	56.2	58.9	70.6	53.3	43.1	56.6	33.3	33.7	4.1
21	90	45	0	87.4	88.3	108.7	72.6	10.0	10.3	45.7	48.2	5.1

22	90	90	45	118.4	180.0	151.2	138.9	10.9	10.3	4.6	4.7	5.8
23	90	135	45	95.4	159.5	115.9	121.2	27.3	25.8	4.2	4.3	5.9
24	90	45	45	117.7	176.3	142.4	130.7	11.8	11.2	4.7	4.8	5.8
25	90	90	90	118.9	127.2	144.2	95.8	5.6	5.5	19.8	20.4	5.5
26	90	135	90	73.6	89.5	90.6	73.0	13.1	13.1	15.9	15.6	4.6
27	90	45	90	101.7	106.9	126.1	84.3	6.0	5.9	20.7	21.4	5.3

APPENDIX B – ENVIRONMENTAL FORCES ON PILE

Here a theoretical method for calculation of wave and current forces on pile have been presented. The linear wave theory was used for calculation of wave components. In the end, comparison was made with Orcaflex results where irregular wave (as per Table 9) was used for analysis for head and beam directions.

The total force (wave and current) exerted on a vertical cylindrical pile [7] is given by

$$dF = \int_{-h}^{0} \frac{1}{2} C_D \rho D u|u| dz + \int_{-h}^{0} C_M \rho \pi \frac{D^2}{4} \frac{Du}{Dt} dz \quad (6)$$

The wave induced velocity and current velocity are combined together [8] and the total force acting on pile is given by

$$dF = \int_{-h}^{0} \frac{1}{2} C_D \rho D \left(v_c + \frac{H\omega}{2} \frac{\cosh(k(h+z))}{\sinh(kh)}\right)^2 \cos(kx - \omega t)\,|\cos(kx - \omega t| dz + \int_{-h}^{0} C_M \rho \pi \frac{D^2}{4} \frac{H\omega^2}{2} \frac{\cosh(k(h+z))}{\sinh(kh)} \sin(kx - \omega t)\, dz \quad (7)$$

The drag force F_{DC} and inertia force F_{IC} constant terms are defined by

$$Let, F_{DC} = \int_{-h}^{0} \frac{1}{2} C_D \rho D \left(v_c + \frac{H\omega}{2} \frac{\cosh(k(h+z))}{\sinh(kh)}\right)^2 dz \quad (8)$$

$$Let, F_{IC} = \int_{-h}^{0} C_M \rho \pi \frac{D^2}{4} \frac{H\omega^2}{2} \frac{\cosh(k(h+z))}{\sinh(kh)} dz \quad (9)$$

F_{DC} is calculated from

$$dF_{DC} = \int_{-h}^{0} \frac{1}{2} C_D \rho D \left({v_c}^2 + 2v_c \frac{H\omega}{2} \frac{\cosh(k(h+z))}{\sinh(kh)} + \frac{H^2\omega^2}{4} \frac{\cosh^2(k(h+z))}{\sinh^2(kh)}\right) dz$$

$$= \frac{1}{2} C_D \rho D \left({v_c}^2 h + v_c H\omega \frac{\sinh(kh)}{k \sinh(kh)} + \frac{H^2 \omega^2}{4 \sinh^2(kh)} \frac{(2kh + \sinh(2kh))}{4k} \right)$$

$$= \frac{1}{2} C_D \rho D \left({v_c}^2 h + \frac{v_c H\omega}{k} + \frac{H^2 gk tanh(kh)}{4 \sinh^2(kh)} \frac{(2kh + \sinh(2kh))}{4k} \right)$$

$$= \frac{1}{2} C_D \rho D \left({v_c}^2 h + \frac{v_c H\omega}{k} + \frac{H^2 g}{2 \sinh(2kh)} \frac{(2kh + \sinh(2kh))}{4} \right) \quad (10)$$

F_{IC} is calculated from

$$F_{IC} = \int_{-h}^{0} C_M \rho \pi \frac{D^2}{4} \frac{H\omega^2}{2} \frac{\cosh(k(h+z))}{\sinh(kh)} dz = C_M \rho \pi \frac{D^2}{4} \frac{H\omega^2}{2k} \quad (11)$$

The total force F as a function of F_{DC} and F_{IC} is given by

$$F = F_{DC} \cos(kx - \omega t) \left|\cos(kx - \omega t)\right| + F_{IC} \sin(kx - \omega t) \quad (12)$$

The maxima of F is calculated from the following equations

$$\frac{dF}{dt} = 0 \quad (13)$$

$$F_{DC}(-2\omega) cos(kx - \omega t)(-\sin(kx - \omega t)) + F_{IC}(-\omega) cos(kx - \omega t) = 0 \quad (14)$$

$$2F_{DC} \sin(kx - \omega t) - F_{IC} = 0 \quad (15)$$

$$\sin(kx - \omega t) = \frac{F_{IC}}{2F_{DC}} \quad (16)$$

The substitution of the value of sin(kx-ωt) in Eq. (12) furnishes the maximum value of F, i.e.

$$F = F_{DC} + \frac{{F_{IC}}^2}{4F_{DC}} \quad (17)$$

The environment parameters used for pile force calculation are shown in Table 20. The wave parameters were derived from *Hs* and *Tp* (Table 9) as per formulations in [9].

Table 20. Environment Parameters

Direction	Hmax	T	k	Current Velocity
Head Sea	1.9	3.6	0.31	2.2
Beam Sea	1.14	3.1	0.42	1.0

The summary of environment forces on the Pile is shown in Table 21. Comparison was also made with irregular wave analysis on a pile model in Orcaflex software. The Linear theory provided higher results and have been considered in the pile design.

Table 21. Pile Forces

Direction	*Fd* (MT)	*FI* (MT)	*Fmax* (MT)	*Fmax_Orcaflex* (MT)
Head Sea	5.7	2.4	6.0	5.2
Beam Sea	1.2	1.4	1.7	1.1